

Sum Rate Maximization in the Uplink of Multi-Cell OFDMA Networks

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Abstract—Resource allocation in orthogonal frequency division multiple access (OFDMA) networks plays an imperative role to guarantee the system performance. However, most of the known resource allocation schemes are focused on maximizing the local throughput of each cell individually, while ignoring the significant effect of inter-cell interference. Thus, the need of considering interference which in itself depends on the allocation of resources (i.e., subcarriers and powers) is evident. In this paper, the joint resource allocation problem via sum rate maximization in multi-cell uplink OFDMA is considered. The problem has a non-convex structure and is known to be NP hard even for single cell scenarios. Initially, upper and lower bounds are investigated. A centralized sub-optimal resource allocation scheme is then developed due to the inherent complexity of implementing the optimal solution. Furthermore, less complex centralized and semi-distributed schemes are proposed. The computational complexity of all schemes has been analyzed and the performance is compared through numerical simulations. Simulation results demonstrate that the semi-distributed scheme achieves comparable performance to the two centralized resource allocation schemes in various scenarios.

I. INTRODUCTION

In this paper, the problem of scheduling (i.e., determining the active users for a given time interval) and power allocation (i.e., assigning powers to these active users) via sum rate maximization in a multi-cell uplink OFDMA network is considered. Solving this problem in an optimal way is cumbersome due to the complexity issues involved in the joint optimization of a large quantity of continuous and discrete variables, non-convexity of the problem even for the continuous power variables, discrete nature of subcarrier allocations and the interdependency of power and subcarrier allocations.

The sum rate maximization problem turned out to be more challenging in the uplink scenario due to the distributed power constraints at each user. Simply allocating subcarrier to the users with best channel quality, the sum-rate may diminish as some active users may have considerably high channel gains but low transmission powers on a specific subcarrier than other competitors. Thus, for uplink, a subcarrier shall be assigned to the users who possess the largest transmission power-channel gain product in order to accomplish near optimal throughput in single cell scenario [1]–[3].

Recently, uplink OFDMA resource allocation via sum rate maximization in single cell scenarios was investigated [1]–[5]. In [1], a near optimal greedy scheme is proposed which is based on maximizing the marginal rate with iterative water-filling on the subcarriers. The authors in [2] generalized the strategy presented in [1] by considering utility maximization and developed a polynomial time algorithm to compute an upper bound of the optimal solution. In [3], sum rate maximization is studied with and without proportional fairness and a transmission power-SNR product (PSP) based ranking scheme is developed; however, the scheme is based on the statistical knowledge of the number of subcarriers assigned to each user. In [4], [5], the authors developed a sub-gradient based scheduling framework to compute the optimal solution of the relaxed problem.

Some interesting papers in multi-cell uplink OFDMA networks

are [6]–[10]. In [6], [7], centralized and distributed schemes are studied which aims at minimizing the overall transmitted power subject to a rate constraint for each user. Nevertheless, most of the work is based on achieving a certain required rate (i.e., fixed rate adaptation) at each user rather than maximizing the network's throughput irrespective of any rate constraint that can exploit the spatial and multiuser diversity implied by the multicell dimensions in the OFDMA network. In [8], intercell coordination is used to maximize the network throughput by removing the cells that do not offer enough throughput; however, OFDMA networks need the use of multiple carriers to exploit the frequency diversity gain. In addition, some recent papers proposed low complexity distributed game theoretic solutions with certain pricing due to the apparent feasibility of implementing them practically in the uplink OFDMA scenario [9], [10]. Nonetheless, all these approaches are suboptimal and no criterion is mentioned to calibrate their performance gap with respect to the optimal solution.

Motivated by this fact, we investigate in this paper sum rate maximization in multi-cell interference-limited uplink OFDMA network subject to an individual power constraint at each user. Initially, we investigate an upper (UB) and lower bound (LB) to the average network throughput. A centralized subcarrier allocation scheme is then developed assuming that a central controller exists possessing global knowledge of channel gains, interfering gains, and user powers in all the cells. Even though the centralized scheme has high computational and implementation cost, it can provide the guiding principles that help in designing low complexity centralized and semi-distributed schemes. In the semi-distributed scheme, we assume that the base stations (BSs) are allowed to cooperate in order to exchange interference information, subcarrier allocation decisions and jointly optimize power allocation in a distributed way.

The rest of the paper is organized as follows: In Section II, the system model is defined and problem is formulated. In Section III, the bounds are derived and their complexity is analyzed. In Section IV and V, the centralized and semi-distributed resource allocation schemes are presented. Section VI demonstrates numerical results followed by concluding remarks in Section VII.

Notation: Throughout the paper, we denote the sets of real and complex vectors of N elements by \mathbb{R}^N and \mathbb{C}^N , respectively. Matrices are represented using boldface upper case letters while bold face lower case letters are used for vectors. $\mathcal{N}(0, \sigma^2)$ denotes a zero mean Gaussian random variable with variance σ^2 .

II. SYSTEM MODEL AND PROBLEM FORMULATION

A network of L cells with a set of K_l users in each cell l is considered. Full reuse of the spectrum is assumed in all the cells, i.e., the reuse ratio is unity. Each BS is assumed to have N orthogonal subcarriers, and each subcarrier can be allocated to a single user per cell. The average throughput per cell is a function of both subcarrier allocation and power allocation variables. The sum rate maximization

problem is formulated using the standard Shannon capacity formula $C_{n,k_l,l} = \log_2(1 + \gamma_{n,k_l,l})$ where $C_{n,k_l,l}$, $\gamma_{n,k_l,l}$ represents the throughput and signal to interference plus noise ratio (SINR) of k_l^{th} user at n^{th} subcarrier in cell l , respectively:

$$\text{maximize}_{\mathbf{P}_l, \mathbf{A}_l} \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(1 + \frac{p_{n,k_l,l} h_{n,k_l,l}}{\sigma^2 + I_{n,l}} \right) \quad (1)$$

$$\text{subject to} \quad \sum_{n=1}^N p_{n,k_l,l} \leq P_{k_l,\max}, \quad \forall k_l, \forall l \quad (2)$$

$$\sum_{k_l=1}^{K_l} \alpha_{n,k_l,l} = 1, \quad \forall n, \forall l \quad (3)$$

$$\alpha_{n,k_l,l} \in [0, 1], \quad \forall n, \forall l, \forall k_l \quad (4)$$

In (1), $I_{n,l} = \sum_{j=1, j \neq l}^L \sum_{k_j=1}^{K_j} \alpha_{n,k_j,j} p_{n,k_j,j} g_{n,k_j,l}$ which represents the cumulative interference at n^{th} subcarrier in cell l from users in all other cells, $p_{n,k_l,l}$ denotes the power transmitted by k_l^{th} user at the n^{th} subcarrier in cell l , $\alpha_{n,k_l,l}$ represents the allocation of k_l^{th} user at the n^{th} subcarrier in cell l and $h_{n,k_l,l}$ is the channel gain of k_l^{th} user at the n^{th} subcarrier in cell l . Constraint (2) implies that the power spent by k_l^{th} user on its allocated subcarriers cannot exceed the maximum power available denoted as $P_{k_l,\max}$. For each cell, we pile the power allocation variables $p_{n,k_l,l}$ into a vector $\mathbf{p}_{n,l} = [p_{n,1}, p_{n,2}, \dots, p_{n,K_l}]$ and then stack all the vectors into a power matrix \mathbf{P}_l of cell l where $\mathbf{P}_l \in \mathbb{R}^{N \times K_l}$. Constraint (3) restricts the allocation of a subcarrier to only one user. The channel gains $h_{n,k_l,l}$ and binary allocation variables $\alpha_{n,k_l,l}$ are stacked up similarly in the matrices \mathbf{H}_l and \mathbf{A}_l , respectively, where $\mathbf{A}_l, \mathbf{H}_l \in \mathbb{R}^{N \times K_l}$. Moreover, we define $g_{n,k_l,l,j}$ as the interfering gain from the k_l^{th} user in cell l to cell j , $\forall j \neq l$ at n^{th} subcarrier. We pile these interfering gains into a vector $\mathbf{g}_{n,l,j} = [g_{n,1,l,j}, g_{n,2,l,j}, \dots, g_{n,K_l,l,j}]$ and then stack all the vectors into a matrix $\mathbf{G}_{l,j} \in \mathbb{R}^{N \times K_l}$. In order to fully exploit the spatial and multiuser diversity gain offered by the multi-cell dimension, no quality of service (QoS) constraints are considered in (1).

III. BOUNDS ON THE NETWORK THROUGHPUT

A. Lower Bound (LB)

The LB for the optimum multi-cell throughput can be achieved when the inter-cell interference becomes maximum. Observing the dependency of inter-cell interference on the subcarrier allocation and power allocation variables, we assume each user in each cell is transmitting on each subcarrier with its maximum power. From (1), the average network throughput can be written as follows:

$$C(\mathbf{A}_l, \mathbf{P}_l) = \frac{1}{L} \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(1 + \frac{p_{n,k_l,l} h_{n,k_l,l}}{\sigma^2 + I_{n,l}} \right) \quad (5)$$

The LB for the average network throughput taking the worst case inter-cell interference into account can be written as follows:

$$C(\mathbf{A}_l, \mathbf{P}_l) \geq \frac{1}{L} \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(1 + \frac{p_{n,k_l,l} h_{n,k_l,l}}{\sigma^2 + \xi_{n,l}} \right) \quad (6)$$

where $\xi_{n,l} = \sum_{j=1, j \neq l}^L \sum_{k_j=1}^{K_j} P_{k_j,\max} g_{n,k_j,l}$. The LB subcarrier allocations \mathbf{A}_l and power allocations \mathbf{P}_l in any cell l can be computed using Algorithm 1 where subcarriers are allocated to the users based on the \mathbf{Q}_l matrices where $Q_{n,k_l,l} = \frac{p_{n,k_l,l} h_{n,k_l,l}}{\xi_{n,l} + \sigma^2}$ is a factor that takes into account the power-worst SINR product of each user at

each subcarrier. These allocations are then utilized to determine the LB network throughput. Since, it has been shown that equal power allocation has insignificant performance loss compared to the optimal water-filling, power equalization among the allocated subcarriers is assumed [1], [3].

B. Upper Bound (UB)

Establishing an upper bound on the uplink throughput in a multi-cell OFDMA network is significantly important in order to calibrate the performance of sub-optimal/near optimal resource allocation schemes. The UB can simply be computed by ignoring the effect of inter-cell interference in all the cells:

$$C(\mathbf{A}_l, \mathbf{P}_l) \leq \frac{1}{L} \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(1 + \frac{p_{n,k_l,l} h_{n,k_l,l}}{\sigma^2} \right) \quad (7)$$

The UB subcarrier allocations \mathbf{A}_l and power allocations \mathbf{P}_l can be computed by substituting $\xi_{n,l} = 0$ in Algorithm 1. These allocations can then be utilized to compute the average network throughput in (7). The near optimality of the average throughput is verified using the UB computed in [2] for the single cell scenario. However, the average network throughput as dictated by the UB allocations could be highly optimistic and may not be accurate for multi-cell systems. Consider a motivating example with two cells, two users and two subcarriers. Each user can transmit with a maximum power of 1W. Assume $\mathbf{H}_1 = [1 \ 0.9; 0.8 \ 0.7]$ and $\mathbf{H}_2 = [1 \ 0.9; 0.8 \ 0.7]$. Single cell allocation strategies that aim to maximize the local throughput of each cell suggest $\mathbf{A}_1, \mathbf{P}_1$ and $\mathbf{A}_2, \mathbf{P}_2 = [1 \ 0; 0 \ 1]$. Computing the UB using (7) results in 1.7655bps/Hz/cell. Now, assuming the knowledge of interfering link gains at each BS, i.e., $\mathbf{G}_{12} = [0.9 \ 0.2; 0.2 \ 0.9]$ and $\mathbf{G}_{21} = [0.7 \ 0.1; 0.1 \ 0.7]$. Computing the throughput again while keeping the single cell allocations and taking into account the interfering gains leads to an average network throughput of 1.1137bps/Hz. However, better allocations are possible if we consider $\mathbf{A}_1, \mathbf{P}_1$ and $\mathbf{A}_2, \mathbf{P}_2 = [0 \ 1; 1 \ 0]$ as per the criterion discussed in Section IV which enhances the resulting average network throughput to 1.5977bps/Hz. The throughput values calculated in the example may not be the maximum possible as the optimal powers are yet to be computed. This issue will be discussed in Section VI.

Algorithm 1 Computing LB and UB Allocations in Cell l

- 1) **Input:** $[\mathbf{H}_l], [\mathbf{A}_l], [\mathbf{P}_l], [\mathbf{G}_{l,j}]$ where $\alpha_{n,k_l,l} = 0, p_{n,k_l,l} = [P_{k_l,\max}/N] \quad \forall k_l, \forall n$
 - 2) For each k_l^{th} user in cell l , power equalization is performed over all the subcarriers allocated previously to that user and the remaining unallocated subcarriers in the system.
 - 3) Using $[\mathbf{P}_l]$ from step 2, $[\mathbf{H}_l]$ and $[\mathbf{G}_{l,j}]$, compute the matrix \mathbf{Q}_l for each cell l .
 - 4) Find the (n, k_l) pair that has the maximum value of $Q_{n,k_l,l}$. Allocate subcarrier n to user k_l .
 - 5) Delete the n^{th} subcarrier from the set of unallocated subcarriers. **If** there are still unallocated subcarriers in the system go to step 2, **else** terminate after distributing the maximum power at each user over all of its assigned subcarriers.
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Complexity Analysis: The (n, k_l) pair at which the $Q_{n,k_l,l}$ becomes maximum is allocated (Step 4), which has a complexity of a two dimensional search, i.e., $O(KN)$. However, as soon as a subcarrier is assigned, each user updates its power as defined in Algorithm 1. This process iterates until all the subcarriers in all the cells are allocated and, thus, the time complexity of Algorithm 1 is $O(KN^2)$.

IV. CENTRALIZED RESOURCE ALLOCATION SCHEMES

Assuming perfect knowledge of channel gains at a centralized controller, the optimal solution for (1) can be computed in the high SINR regime by an exhaustive search over all possible combinations of the allocations in each cell. For each possible allocation, optimum powers can be derived by transforming (1) into a geometric program (GP). Importantly, the power allocation problem is in itself a known non-convex problem for the general SINR regime. However, for the high SINR regime the problem turns into a convex GP problem. For a given set of allocation variables and considering a high SINR regime, the objective function in (1) can be rewritten as follows:

$$\underset{\mathbf{P}_l}{\text{maximize}} \quad \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(\frac{p_{n,k_l,l} h_{n,k_l,l}}{\sigma^2 + I_{n,l}} \right) \quad (8)$$

Maximizing the SINRs is equivalent to minimizing the interference to signal ratio (ISNR), the objective can thus be rewritten as

$$\underset{\mathbf{P}_l}{\text{minimize}} \quad \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(\frac{\sigma^2 + I_{n,l}}{p_{n,k_l,l} h_{n,k_l,l}} \right) \quad (9)$$

Equivalently, (1) can be reformulated for high SINR regime as follows:

$$\begin{aligned} \underset{\mathbf{P}_l}{\text{minimize}} \quad & \log_2 \prod_{l=1}^L \prod_{k_l=1}^{K_l} \prod_{n=1}^N \left(\frac{\sigma^2 + I_{n,l}}{p_{n,k_l,l} h_{n,k_l,l}} \right)^{\alpha_{n,k_l,l}} \\ \text{subject to} \quad & \sum_{n=1}^N p_{n,k_l,l} \leq P_{k_l,\max}, \quad \forall k_l, \forall l \end{aligned} \quad (10)$$

Note that the numerator in (10) is a posynomial and the denominator is a monomial, hence (10) is a GP problem in standard form that can be solved optimally through efficient interior point methods [11] after performing the logarithmic transformation of variables [12]. However, even for small dimensions, it is not recommendable to compute the optimal solution due to the huge computational complexity of $O(K^{LN})$ associated with an exhaustive search based subcarrier allocation phase. In addition, GP based power allocation method discussed above has two restrictions: high-SINR assumption and centralized time-consuming computations. These limitations, however, can be possibly reduced through the heuristics proposed in [12]. Considering the high intricacy of implementing the optimal solution, a two-stage centralized scheme is developed. Moreover, a second centralized scheme with reduced complexity is discussed.

A. Centralized Scheme A

In the uplink, the subcarrier allocation and power allocation phases cannot be detached since subcarrier allocation depends on the powers constraints at every user. In Phase I, we define the term $\chi_{n,k_l,l} = \frac{p_{n,k_l,l} h_{n,k_l,l}}{\sum_{j=1, j \neq l}^L P_{k_l,\max} g_{n,k_l,l,j}}$ for the allocation of resources to users. This criterion guarantees the selection of the users who have not only better power-gain product but also result in less interference to the neighbour cells as the denominator $\sum_{j=1, j \neq l}^L P_{k_l,\max} g_{n,k_l,l,j}$ accounts for the maximum interference that the k_l^{th} user may cause to cell j . The details are provided in Algorithm 2. Once the subcarrier allocation is performed, the optimal powers can be calculated for the high SINR regime as explained above or for the general SINR regime through solving a series of GPs using successive convex approximation which is a provably convergent heuristic [12]. This approach is known to compute globally optimal power allocations in many cases. Nevertheless, like any sub-optimal scheme, the convergence and optimality of the power allocation phase in the general

SINR regime depends on the initialization of the power allocation variables. However, we observe that fast convergence can be achieved by using the optimal powers computed in the high SINR regime for initialization. The details of the implementation can be found in the extended journal version [13] of this paper.

Algorithm 2 Centralized Resource Allocation for L Cells

- 1) **Input:** $[\mathbf{H}_l], [\mathbf{A}_l], [\mathbf{P}_l], [\mathbf{G}_{l,j}]$ where $\alpha_{n,k_l,l} = 0, p_{n,k_l,l} = [P_{k_l,\max}/N] \quad \forall k_l, \forall n$
Subcarrier Allocations (Phase I)
 - 2) Compute $\chi_{n,k_l,l}$ for every k_l^{th} user at n^{th} subcarrier in cell l . $n = 1$, **do while** $n \leq N, n = n + 1$
 - 3) Find the (n, k_l, l) pair with maximal value of $\chi_{n,k_l,l}$.
 - 4) Perform allocations at n^{th} subcarrier in the remaining cells $j, \forall j \neq l$ by searching for the user with maximum value of $\chi_{n,k_l,j}$.
 - 5) Perform power update (equalization) as detailed in Algorithm 1 in each cell l .
 - 6) Remove the subcarrier n from the set of unallocated subcarriers. **end**
 - Power Allocations (Phase II)**
 - 7) Compute the optimal powers \mathbf{P}_l in the high SINR regime using (10) given the allocations from Phase I.
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Complexity Analysis: Initially we perform a three dimensional search over $\chi_{n,k_l,j}$ which has a complexity of $O(LKN)$. Next, a linear search is conducted in the cells other than the starting cell with a complexity of $O((L-1)K)$. The process iterates until all subcarriers are allocated, thus the total complexity of Phase I is $O(LKN^2 + NK(L-1))$. The complexity of Phase II is difficult to determine, however, it can be measured in terms of degree of difficulty (DoD) that in turn relies on the number of constraints and variables associated with the GP [14]. Since we are dealing with LK power constraints and LKN power variables, apparently it seems that implementing centralized GP/successive GP based schemes may not be a good choice for practical implementations. However, in order to reduce the complexity and DoD of GP we have developed the following less complex centralized scheme.

B. Centralized Scheme B

As the time complexity of Phase I is delectable compared to Phase II, in this approach, instead of splitting the whole procedure into two phases we combine them in the following manner. At first, the allocations are performed in each cell l followed by the power update (based on equalization) at subcarrier n as mentioned in Algorithm 2 (Steps 2-5). Before performing the next subcarrier allocation, we compute GP based optimal power allocations on subcarrier n as the powers dictated by the equalization process $p_{n,k_l,l_{\text{eq}}}$ may not be optimal. Setting the powers $p_{n,k_l,l_{\text{eq}}}$ as the upper bound on the power variable of k_l^{th} user in each cell l at subcarrier n , we now define the less complex GP problem in order to compute the optimal powers that can maximize the throughput at n^{th} subcarrier.

$$\begin{aligned} \underset{p_{n,k_l,l}}{\text{minimize}} \quad & \log_2 \prod_{l=1}^L \left(\frac{\sigma^2 + I_{n,l}}{p_{n,k_l,l} h_{n,k_l,l}} \right) \\ \text{subject to} \quad & p_{n,k_l,l} \leq p_{n,k_l,l_{\text{eq}}}, \quad \forall l \end{aligned} \quad (11)$$

Clearly, optimal power of each user at subcarrier n may not succeed in reaching the upper bound. Hence, this unused proportion of power that a user cannot use in reaching the upper bound, must be taken into account while doing the next round of allocations.

Thus, this unused proportion of power is distributed among the remaining unallocated subcarriers of that user. The DoD reduces to L constraints and variables. Although this procedure restricts the degree of freedom offered by GP, numerical results show that the network throughput remains comparable with reduced complexity. The procedure is detailed in Algorithm 3.

Algorithm 3 Low Complexity Centralized Resource Allocation for L Cells

- 1) Compute $\chi_{n,k_l,l}$ for every k_l^{th} user at n^{th} subcarrier in cell l . $n = 1$, **do while** $n \leq N$, $n = n + 1$
 - 2) Steps 2-5 of Algorithm 2.
 - 3) Compute the optimal powers $\mathbf{p}_{n,k_l,l}$ in the high SINR regime using (11).
 - 4) Perform power equalization of k_l^{th} user among the remaining unallocated subcarriers.
 - 5) Remove the subcarrier n from the set of unallocated subcarriers. **end**
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V. SEMI-DISTRIBUTED RESOURCE ALLOCATION SCHEME

We assume each BS knows the maximum interference caused by its users to the first tier of interfering cells. Assuming the knowledge of interfering gains here rendered this approach semi-distributed; however, we present in [13] a more distributed scheme that does not require the sharing of interfering gains among BSs. In the proposed semi-distributed scheme, each BS searches for the user with maximum value of $\chi_{n,k_l,l}$ in order to perform allocations. The allocation decisions are locally made at each BS (Phase I) and do not depend on the resource allocations and powers in the neighbour interfering cells. Once the allocations are decided each cell shares them with all other interfering cells. The optimal distributed power allocations can then be computed using dual decomposition methods by first performing the log transformation of the variables, i.e., $\ln p_{n,k_l,l} = p_{n,k_l,l}$ and $\ln p_{n,k_j,j} = p_{n,k_j,j}$, then adding auxiliary variable $z_{n,lj} = \sum_{j=1, j \neq l}^L h_{n,k_j,l} p_{n,k_j,j}$ in order to transfer the coupling in the objective to coupling into the constraints [12]. This reduces the complexity of implementing GP by decoupling the primal problem into L dual problems, i.e., one for each cell. The problem can be then be written as follows:

$$\begin{aligned}
 & \underset{\mathbf{z}, \mathbf{p}}{\text{minimize}} && \sum_{l=1}^L \sum_{k_l=1}^{K_l} \sum_{n=1}^N \alpha_{n,k_l,l} \log_2 \left(\frac{\sigma^2 + e^{z_{n,lj}}}{p_{n,k_l,l} h_{n,k_l,l}} \right) \\
 & \text{subject to} && \sum_{n=1}^N \alpha_{n,k_l,l} p_{n,k_l,l} \leq P_{k_l, \max}, \quad \forall k_l, \forall l \quad (12) \\
 & && e^{z_{n,lj}} \geq \sum_{j=1, j \neq l}^L h_{n,k_j,l} e^{p_{n,k_j,j}}, \quad \forall n
 \end{aligned}$$

Writing the Lagrangian $L(p_{n,k_l,l}, z_{n,lj}, \lambda_{k_l}, \eta_{n,lj})$ for (12) and splitting the problem into L sub-problems yield (13) with local variables $p_{n,k_l,l}, z_{n,lj}, \lambda_{k_l}$ and coupling variables $\eta_{n,lj}$. The details of the solution are omitted here due to space limitation but presented in the journal version of this paper [13].

$$\max \min L(p_{n,k_l,l}, z_{n,lj}, \lambda_{k_l}, \eta_{n,lj}) \quad (13)$$

VI. SIMULATION RESULTS AND PERFORMANCE EVALUATION

A cellular OFDMA network is considered where the radius of each cell is assumed to be $R_c = 0.5\text{km}$. The users lie at equally spaced angles from 0 to 2π . For demonstration purpose, the total number

of users are assumed to be same in all the cells. The maximum user transmit power is considered to be 2W. The channel gain is defined as $h_{n,k_l,l} = (-122 - 10\gamma \log_{10} d_{k_l,l}) - \mathcal{N}(0, \sigma^2) + 10 \log_{10} F_{n,k_l,l}$. The first term denotes the path loss where γ is the path loss exponent and is set equal to 3.8. The second term represents log-normal shadowing with a mean of 0 dB and a standard deviation of 8 dB. The last factor, $F_{n,k_l,l}$ corresponds to Rayleigh fading. The bandwidth of the system is assumed to be 5 MHz with a noise power spectral density of $2.07 \times 10^{-14} \text{W/Hz}$ at each receiver. The channel conditions are assumed to be fixed during a frame. The interfering gains from j^{th} interfering cell to the cell of interest l are computed as follows $g_{n,k_j,l} = (-122 - 10\gamma \log_{10} d_{k_j,l}) - \mathcal{N}(0, \sigma^2) + 10 \log_{10} F_{n,k_j,l}$.

In Table 1, we compare the maximum achievable average network throughput (measured in bps/Hz) and complexity of the proposed centralized and semi-distributed schemes with the derived bounds and the optimal solution in the high SINR regime. The optimal solution is computed via an exhaustive search based subcarrier allocation phase detailed in Section IV. The results are taken after averaging over 100 channel realizations. It is observed that the performance gap of the centralized strategies and the optimal solution is delectable. However, this outcome may not remain valid for higher dimensions. In addition, the less complex centralized scheme B provides comparable results to centralized scheme A. It is also worth to mention here that in order to get a more tighter LB, we compute the LB throughput in Fig.1 and Fig.2 by computing the optimal power allocations based on GP given the LB allocations from Algorithm 1. Next, we compare the performance of the centralized scheme B and the semi-distributed scheme with the UB and LB for varying number of users and subcarriers in 2-cell (See [13]), 4-cell and 7-cell scenario. The results have been taken after averaging over 1000 channel realizations. We observe that for the 2-cell case, the centralized and semi-distributed strategies give nearly similar results, however, as the number of cell increases the performance gain of centralized scheme over semi-distributed scheme is evident (see Fig.1 and Fig.2). The UB results remain independent to the number of cells, however, it is clear from Fig.1 and Fig.2 that the LB throughput continues to decrease with the increase in number of interfering cells. Moreover, it can be easily observed that the performance gap between the centralized and semi-distributed strategies tends to increase with the increase in number of subcarriers, users and interfering cells. The established UB and LB provide a comprehensive idea of a finite performance gap.

Another important observation in GP based optimal power allocation is that the sum $\sum_{n=1}^N p_{n,k_l,l}$ may or may not be equal to $P_{k_l, \max}$ as dictated by the equalization based power allocation. Moreover, it may not (almost never) be optimal that two users transmit with powers suggested by power equalization based power allocations. Consider a channel realization $\mathbf{H}_1, \mathbf{H}_2 = [0.3028 \ 0.2503; 0.0388 \ 0.1529] \times 10^{-9}$ and $\mathbf{G}_{12}, \mathbf{G}_{21} = [0.0597 \ 0.0455; 0.1555 \ 0.0597] \times 10^{-11}$ and $[0.1340 \ 0.6860; 0.7538 \ 0.1935] \times 10^{-11}$, respectively. The equal power allocations dictates $\mathbf{P}_1 = [0 \ 0.5; 0 \ 0.5]$ and $\mathbf{P}_2 = [0.5 \ 0; 0.5 \ 0]$ for semi-distributed scheme that leads to the average throughput of 11.8392bps/Hz. However, computing the optimal powers results in $\mathbf{P}_1 = [0.0000 \ 0.5326; 0.0000 \ 0.4674]$ and $\mathbf{P}_2 = [0.3840 \ 0.0000; 0.6160 \ 0.0000]$ which lead to a maximum average network throughput of 17.2734bps/Hz. This example demonstrates the significance of GP as well as centralized scheme B in which after allocating each subcarrier, optimal powers are computed and thus, the remaining unused power can be accounted for while computing the next allocations. The significance of GP is also demonstrated in Fig. 3, where we compare the performance of the semi-distributed scheme with and without GP based power

TABLE I
AVERAGE NETWORK THROUGHPUT OF THE CENTRALIZED/SEMI-DISTRIBUTED SCHEMES FOR $L=2$ CELLS AND $N = 6$ SUBCARRIERS/CELL

	$K=2$		$K=4$		$K=6$		Complexity
	$d_{i,k_l,l}=0.35\text{km}$	$d_{i,k_l,l}=0.45\text{km}$	$d_{i,k_l,l}=0.35\text{km}$	$d_{i,k_l,l}=0.45\text{km}$	$d_{i,k_l,l}=0.35\text{km}$	$d_{i,k_l,l}=0.45\text{km}$	
UB	52.6264	30.5791	63.5723	40.4159	69.1927	46.1368	$O(KN^2)$
Optimal	39.3301	20.0545	49.6299	28.4265	55.6299	34.8121	$O(K^{NL}) + DoD(LKN)$
Centralized A	38.6454	19.2522	48.7848	27.5679	54.5671	33.8055	$O(LKN^2 + (L-1)KN) + DoD(LKN)$
Centralized B	38.5254	19.0222	48.5248	27.5479	54.3671	33.7925	$O(LKN^2 + (L-1)KN) + DoD(L)$
Semi-distributed	37.9408	18.7190	48.0243	27.5107	54.5178	33.7270	$O(KN^2) + DoD(KN)$
LB	35.2340	16.3582	43.6006	22.3586	48.3627	27.4565	$O(KN^2)$

allocations. Without GP, the resulting throughput is observed to be lowered significantly than the centralized scheme.

VII. CONCLUSION

In this paper, we developed an UB and LB to the problem of sum rate maximization in multi-cell uplink OFDMA networks. Moreover, we proposed two centralized and one semi-distributed resource allocation schemes. All schemes are evaluated and compared in terms of network throughput and computational complexity.

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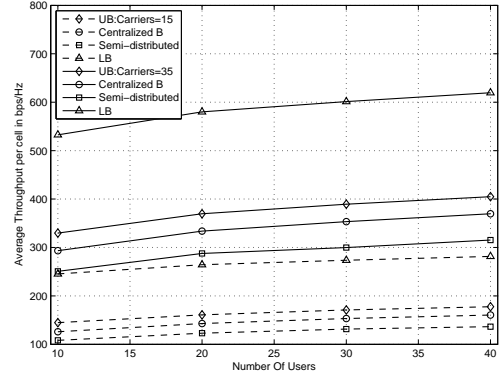


Fig. 1. Comparison of the centralized B/semi-distributed schemes for $L=7$ cell, $d=0.4\text{km}$

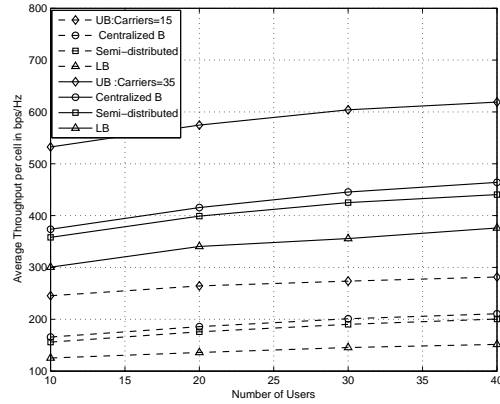


Fig. 2. Comparison of the centralized B/semi-distributed schemes for $L=4$ cell, $d=0.4\text{km}$

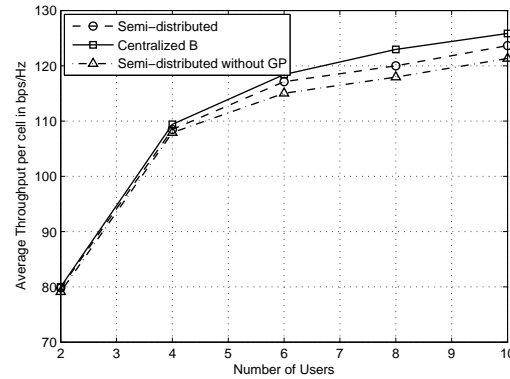


Fig. 3. Significance of GP based optimal power allocations in high SINR regime for $L=2$ cells, $N=10$ subcarriers/cell and $d=0.4\text{km}$: An example